MATH 476 Statistics

Fred J. Hickernell — Test 1 Make Up — Thursday, February 19, 2026

Instructions:

i. *This test has* ***FOUR*** *question(s). Attempt all. The maximum number of points is* ***100****.*

ii. *The time allowed is 75 minutes.*

iii. *This test is closed book, but you may use 4 double-sided letter-size sheets of notes.*

iv. *No calculators or other electronic devices are allowed. Phones must be placed in your bags under your desks or at the front of the room. Hands must be on top of your desks.*

v. *You are expected to simplify any expressions that can be simplified as whole numbers, e.g.,* $36/15 = 12/5$*, but you may leave other expressions as is, e.g.,* $\sqrt{47}$ *or* $18/29$.

vi. *You will be provided all of the critical values that you need.*

vii. *Show all your work to justify your answers. Answers without adequate justification will not receive credit.*

viii. *Print your name clearly on each answer sheet.*

ix. *Off-site students may contact the instructor as directed by your syllabus.*

I understand these instructions and have not relied on any help for this exam beyond what is allowed, nor provided any help to anyone else beyond what is allowed.

______________________________ ____________________
Signature *Date*

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Printed Name

Upper quantiles: q_α **satisfies** $\mathbb{P}(X > q_\alpha) = \alpha$

X	α	0.995	0.99	0.975	0.95	0.9	0.1	0.05	0.025	0.01	0.005
Norm(0, 1)	z_α	-2.58	-2.33	-1.96	-1.64	-1.28	1.28	1.64	1.96	2.33	2.58
t_ν	$t_{14,\alpha}$	-2.98	-2.62	-2.14	-1.76	-1.35	1.35	1.76	2.14	2.62	2.98
	$t_{15,\alpha}$	-2.95	-2.60	-2.13	-1.75	-1.34	1.34	1.75	2.13	2.60	2.95
	$t_{16,\alpha}$	-2.92	-2.58	-2.12	-1.75	-1.34	1.34	1.75	2.12	2.58	2.92

1. (30 points) Given one observation, X, of an $\mathrm{Exp}(\lambda)$ random variable. Construct a 95% confidence interval for λ.

Answer: Since X has a distribution, $F(x) = 1 - \exp(-\lambda x)$, we can say that

$$95\% = \mathbb{P}(x_{0.975} \leq X \leq x_{0.025}),$$

where x_α denotes the upper quantile:

$$\alpha = \mathbb{P}(X > x_\alpha) = \exp(-\lambda x_\alpha) \implies x_\alpha = \frac{\log(1/\alpha)}{\lambda}.$$

Thus,

$$95\% = \mathbb{P}\left(\frac{\log(1/0.975)}{\lambda} \leq X \leq \frac{\log(1/0.025)}{\lambda}\right) = \mathbb{P}\left(\frac{\log(1/0.975)}{X} \leq \lambda \leq \frac{\log(1/0.025)}{X}\right)$$

and the confidence interval is

$$\left[\frac{\log(1/0.975)}{X}, \frac{\log(1/0.025)}{X}\right].$$

2. (25 points) Given $\mathrm{N}(\mu, \sigma^2)$ IID data, construct a 95% confidence interval for μ based on $n = 16$ samples with an observed $\overline{x} = 15$ and $s^2 = 25$.

Answer: Because the sample size is small, but the data is normal, we may use the Student's t distribution:

$$\begin{aligned}\frac{\overline{X}_n - \mu}{S/\sqrt{n}} \sim t_{n-1} \implies 95\% &= \mathbb{P}\left(\mu - t_{n-1,0.025}\frac{S_n}{\sqrt{n}} \leq \overline{X}_n \leq \mu + t_{n-1,0.025}\frac{S_n}{\sqrt{n}}\right) \\ &= \mathbb{P}\left(\overline{X}_n - t_{n-1,0.025}\frac{S_n}{\sqrt{n}} \leq \mu \leq \overline{X}_n + t_{n-1,0.025}\frac{S_n}{\sqrt{n}}\right)\end{aligned}$$

So the 95% confidence interval based on this data is

$$\overline{x}_{16} \pm t_{15,0.025}\frac{s_{16}}{\sqrt{16}} = 15 \pm 2.13 \times \frac{5}{4} = 15 \pm 2.13 \times 1.25 = 15 \pm 2.66$$

3. (30 points) If your chance of making a free throw is 40%,

a. (15 points) What is the median number of free throws that you need to attempt before making your first one?

Answer: The media is defined as $Q(1/2)$*, where*

$$Q(p) = \inf\{x : F(x) \geq p\}.$$

If X *is the number of free throws that you need to make before making the first one, then*

$$\begin{aligned} \varrho(1) &= \textit{probability of making your first one} = 0.4, \\ F(1) &= \varrho(1) = 0.4 \\ \varrho(2) &= \textit{probability of missing your first and making your second} = 0.6 \times 0.4 = 0.24 \\ F(2) &= \varrho(1) + \varrho(2) = 0.64 \end{aligned}$$

So the median is 2*.*

b. (15 points) Compute the conditional probability mass function of making T out of 4 free throws given that you made X out of your first 2.

Answer:

$\varrho_{T\mid X}(t\mid x)$	$t =$	0	1	2	3	4
$x =$	0	$0.6 \times 0.6 = 0.36$	$2 \times 0.4 \times 0.6 = 0.48$	$0.4 \times 0.4 = 0.16$	0	0
	1	0	0.36	0.48	0.16	0
	2	0	0	0.36	0.48	0.16

4. (15 points) If you perform a pilot experiment and compute a 95% confidence interval for the mean with $n = 50$ and a half-width of 0.35 using a large sample approximation, about how large would you need to make n to get a half-width of 0.07?

Answer: Let h_n *denote the half width*

$$\begin{aligned} 0.35 = h_{50} \approx 1.96\frac{\sigma}{\sqrt{50}} &\implies \sigma = \frac{0.35\sqrt{50}}{1.96} \\ 0.07 = h_n \approx 1.96\frac{\sigma}{\sqrt{n}} &\implies n \approx \left(\frac{1.96\sigma}{0.07}\right)^2 = \left(\frac{1.96 \times 0.35\sqrt{50}}{0.07 \times 1.96}\right)^2 = 5^2 \times 50 = 1250 \end{aligned}$$