MATH 476 Statistics

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Test **Monday, April 12**

Instructions:

i. *This test consists of* THREE *questions. Answer all of them.*

ii. *The time allowed for this test is 75 minutes.*

iii. *The data and situations portrayed in this test are fictitious, but realistic.*

iv. *This test is closed book, but you may use 2 double-sided letter-size sheets of notes.*

v. *Calculators, even of the programmable variety, are allowed. Computers using JMP or MATLAB, are also allowed. No internet access, web browsing, email, chat, etc. is allowed.*

vi. *Show all your work to justify your answers. Answers without adequate justification will not receive credit.*

Sarita and Shlomo are two consultants employed by ACME Widget Manufacturers. You are the chief statistician at ACME and are supervising Sarita and Shlomo.

1. (40 marks)
The order processing department claims to make errors on no more than 2% of the orders. Let θ denote the proportion of errors. The null hypothesis is $H_0 : \theta = 2\%$, and the alternative hypothesis is $H_a : \theta > 2\%$. You ask Sarita and Shlomo to design a hypothesis test. Sarita proposes to select 3 orders at random and reject H_0 if and only if 1 or more errors are detected. Shlomo proposes to select 20 orders at random and reject H_0 if and only if 2 or more errors are detected.

   a) Derive the Type I error, α, for each of these two tests.

   Answer: The probability of rejecting H_0 under Sarita's test if H_0 is true is

$$\begin{aligned}\alpha &= \Pr(1 \textit{ or more errors out of } 3 \textit{ orders} | \theta = 2\%) \\ &= 1 - \Pr(\textit{no errors out of } 3 \textit{ orders} | \theta = 2\%) = 1 - 0.98^3 = 5.88\%.\end{aligned}$$

   The probability of rejecting H_0 under Shlomo's test if H_0 is true is

$$\begin{aligned}\alpha &= \Pr(2 \textit{ or more errors out of } 20 \textit{ orders} | \theta = 2\%) \\ &= 1 - \Pr(0 \textit{ or } 1 \textit{ errors out of } 20 \textit{ orders} | \theta = 2\%) \\ &= 1 - 0.98^{20} - 20 \times 0.98^{19} \times 0.02 = 5.99\%,\end{aligned}$$

   which is nearly the same.

   b) Derive the Type II error, β, for each of these two tests as a function of θ.

   Answer: The probability of not *rejecting H_0 under Sarita's test if H_a is true is*

$$\beta = \Pr(\textit{no errors out of } 3 \textit{ orders} | \theta) = (1-\theta)^3, \quad \theta > 2\%.$$

   The probability of rejecting H_0 under Shlomo's test if H_0 is true is

$$\begin{aligned}\beta &= \Pr(0 \textit{ or } 1 \textit{ errors out of } 20 \textit{ orders} | \theta) \\ &= (1-\theta)^{20} + 20\theta(1-\theta)^{19}, \\ &= (1-\theta)^{19}(1+19\theta), \quad \theta > 2\%.\end{aligned}$$

c) If the true error rate is $\theta = 10\%$, what are the probabilities of each of these two tests *not* rejecting the null hypothesis?

Answer: For Sarita's test the probability is

$$(1-\theta)^3 = 0.9^3 = 72.90\%,$$

and for Shlomo's test the probability is

$$(1-\theta)^{19}(1+19\theta) = 0.9^{19}(2.9) = 39.17\%.$$

d) As the chief statistician, what would you advise the CEO of ACME Widgets are the pros and cons of these two tests.

Answer: Sarita's test is less costly to perform but much less powerful than Shlomo's test.

2. (30 marks)
Sarita inspects the lifetimes of widgets produced in the plants in Aurora, Batavia, and Cicero (see below). All plants are supposed to be producing widgets with the same mean lifetimes. Does the data bear this out? Report the p-value of the test used along with the conclusion.

Plant	Widget Lifetimes (in hours)									
Aurora	20	32	14	31	27	26	20	15	31	26
Batavia	30	16	25	24	28	22	18	30		
Cicero	19	15	20	13	17	16	19	20	23	

Answer: Input all the widget lifetimes in one column in JMP. Label them by factory in the second column. Choose `Fit Y by X` *to do a oneway ANOVA. This tests the null hypothesis of all widgets produced by the three factories having the same average lifetimes. The p-value is* 2.8%*, which leads one to reject the null hypothesis and conclude that the mean lifetimes from the three factories are* not *all the same.*

3. (30 marks)
The VP of Sales observes that customers from different geographical areas have preferences for different models of widgets. Shlomo randomly surveys 1000 customers out of the hundreds of thousands and finds the following preferences:

	Model Preference		
Region	Z-100	Z-200	Z-300
East Coast	123	115	95
Midwest	111	109	117
West Coast	133	108	89

Does this data indicate a difference in model preferences among the regions or not?

Answer: In JMP put all three copies of the three region names in a column labeled `Regions`*. In the second column, labeled* `Model Preference` *put copies of the three different models so that there is one pair of region and model preference for every possible choice. In the third column, labeled* `Frequency` *input the frequencies. Then choose* `Analyze/Fit Y By X` *and put* `Brand Preference` *into* `Y`*,* `Region` *into* `X`*, and* `Frequency` *into* `Frequency` *and submit. Since the* $p = 17\%$*, there is no compelling reason to reject the null hypothesis of no difference in model preferences across regions.*