MATH 332 Linear Algebra: Theory and Applications

Fred J. Hickernell Quiz 4 Thursday, November 12, 2026

Instructions:

i. *This quiz has **three questions**, worth **20 points**.*

ii. *The time allowed is **15 minutes**. Approved testing accommodations apply.*

iii. *This quiz is closed book, but you may use up to **4** double-sided letter-sized sheets of notes.*

iv. *No calculators or other electronic devices are allowed. Silence and put devices in your bag under your desk or at the front of the room.*

v. *Work individually. Do not communicate with other students or share materials. Direct questions to the instructor.*

vi. *Show your work and give a brief justification for each answer.*

vii. *Print your name clearly on each answer sheet.*

Q	Max	Score
1	6	
2	4	
3	10	
Tot	**20**	

I understand these instructions and have not relied on any help for this exam beyond what is allowed, nor provided any help to anyone else beyond what is allowed.

_____ _____

Signature *Date*

_____

Printed Name

Let

$$\boldsymbol{u} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \qquad \boldsymbol{v} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}, \qquad V = \operatorname{span}\{\boldsymbol{u}, \boldsymbol{v}\} \subseteq \mathbb{R}^3.$$

Use these vectors and this vector space for all three questions.

1. (6 points) Is $\mathcal{B} = \{\boldsymbol{u}, \boldsymbol{v}\}$ a basis of V? If not, redefine $\mathcal{B}$ to make it a basis of V.

   Answer: Yes. The vectors span V by its definition. To check independence, row-reduce the augmented matrix for $a\boldsymbol{u} + b\boldsymbol{v} = \boldsymbol{0}$:

$$\left[\begin{array}{cc|c} 1 & 2 & 0 \\ 2 & 1 & 0 \\ -1 & 1 & 0 \end{array}\right] \xrightarrow[R_3 \leftarrow R_3 + R_1]{R_2 \leftarrow R_2 - 2R_1} \left[\begin{array}{cc|c} 1 & 2 & 0 \\ 0 & -3 & 0 \\ 0 & 3 & 0 \end{array}\right] \xrightarrow{R_3 \leftarrow R_3 + R_2} \left[\begin{array}{cc|c} 1 & 2 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 0 \end{array}\right].$$

   The second row gives $b = 0$, and then the first gives $a = 0$. Thus the vectors are linearly independent. They therefore form a basis of V; no redefinition is needed.

2. (4 points) What is $\dim V$? Does $V = \mathbb{R}^3$?

Answer: $\boxed{\dim V = 2}$, *because the basis* $\mathcal{B}$ *contains two vectors. No,* $\boxed{V \neq \mathbb{R}^3}$, *since* $\dim \mathbb{R}^3 = 3$.

3. (10 points) Let $\boldsymbol{w} = \begin{bmatrix} 1 \\ 5 \\ -4 \end{bmatrix}$. Does $\boldsymbol{w}$ belong to V? If so, find its coordinate vector in terms of the basis $\mathcal{B}$ that you identified in Question 1. Use the order in which you listed the basis vectors in Question 1.

Answer: Solve $a\boldsymbol{u} + b\boldsymbol{v} = \boldsymbol{w}$ *by reducing its augmented matrix to echelon form:*

$$\left[\begin{array}{cc|c} 1 & 2 & 1 \\ 2 & 1 & 5 \\ -1 & 1 & -4 \end{array}\right] \xrightarrow[R_3 \leftarrow R_3 + R_1]{R_2 \leftarrow R_2 - 2R_1} \left[\begin{array}{cc|c} 1 & 2 & 1 \\ 0 & -3 & 3 \\ 0 & 3 & -3 \end{array}\right] \xrightarrow{R_3 \leftarrow R_3 + R_2} \left[\begin{array}{cc|c} 1 & 2 & 1 \\ 0 & -3 & 3 \\ 0 & 0 & 0 \end{array}\right].$$

The system is consistent: the last row is $0 = 0$. *Use back substitution. The second row gives* $-3b = 3$, *so* $b = -1$. *Substituting into the first row gives* $a + 2(-1) = 1$, *so* $a = 3$. *Thus* $\boldsymbol{w} = 3\boldsymbol{u} - \boldsymbol{v}$. *Hence* $\boldsymbol{w} \in V$ *and* $\boxed{[\boldsymbol{w}]_{\mathcal{B}} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}}$. *These two coordinates are the coefficients in the listed basis order.*

Quiz 4 Scores (out of 20)
Number of Students: 17, Minimum: 0, Maximum: 20, Mean: 12.4, Median: 13
Standard Deviation: 7.6, Quartiles (Q1, Q3): (7, 20)

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2|0000000
1|0133
0|001778
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Quite a few did perfectly and quite a few struggled. These are questions about subspaces of vector spaces that should become straightforward to you.